

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5540 Advanced Geometry 2016-2017

Assignment 4

Due Date: 27 Apr, 2017

1. Recall that the Poincaré metric on the Poincaré disk \mathbb{D} is $ds^2 = \frac{4(dx^2 + dy^2)}{(1 - x^2 - y^2)^2}$.

Suppose that $\gamma(t) = (x(t), y(t)) = (\frac{t}{2}, \frac{1}{2})$, for $0 \leq t \leq 1$, is a curve on \mathbb{D} . Find the length of γ with respect to the Poincaré metric.

2. Let $z \in \mathbb{D}$ and $|z| = r$. Show directly that the distance $d(0, z)$ between 0 and z with respect to the Poincaré metric is $\ln \left(\frac{1+r}{1-r} \right)$.

3. Let $z_1 = \frac{i}{2}$ and $z_2 = \frac{1}{2} + \frac{i}{2}$ be two points on \mathbb{D} .

(a) Find the equation of P -line passing through z_1 and z_2 and express your answer in form of $(x - h)^2 + (y - k)^2 = r^2$.

(b) Find the distance $d(z_1, z_2)$ between z_1 and z_2 with respect to the Poincaré metric. (Remark: Compare to the answer obtained in question 1)

4. Let $A = z_1 = \frac{1}{2} + \frac{i}{2}$, $B = z_2 = \frac{i}{2}$ and $C = z_3 = \frac{1}{2} - \frac{i}{2}$ be three points on \mathbb{D} .

(a) Find the P -angle $\angle BAC$.

(Hint: Consider a suitable $f \in \text{Aut}(\mathbb{D})$ such that the image of A under f is 0.)

(b) Find the sum of interior P -angles of the P -triangle $\triangle ABC$, does it less than 180° ?

5. Let $A = \frac{1}{2} - \frac{i}{2}$ and $C = \frac{i}{2}$. Find the equation of P -circle centered at C with P -line segment CA as radius.

6. Let Γ be a P -circle given by the equation $|z - \frac{1}{2}| = \frac{1}{4}$. Find the P -center of Γ .

(Hint: $z_1 = \frac{3}{4}$ and $z_2 = \frac{1}{4}$ are points lying on Γ and the P -line segment joining them is a diameter.)